Computing with characteristic functions: From numerical inversion to multivariate logistic goodness-of-fit

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Abstract

Characteristic functions provide a natural computational interface between probability models and statistical inference, especially when densities or distribution functions are unavailable in closed form or are numerically cumbersome. This lecture first gives a brief overview of characteristic-function computation and numerical inversion as implemented in the MATLAB CharFunTool [**?**], including numerical algorithms for univariate and bivariate distributional calculations [**?**].

These computational tools provide a natural basis for extending characteristic-function methods from probability-distribution evaluation to statistical inference based on empirical characteristic functions.

The main part is devoted to a recently developed goodness-of-fit test for multivariate logistic distributions [**?**], illustrating how the computational methodology underlying CharFunTool can be extended from distributional calculations to statistical inference. After affine standardization of the observations, the test compares the empirical characteristic function with the theoretical characteristic function of the standard multivariate logistic model through a weighted L^2 distance. Its implementation combines numerical characteristic-function calculations and Monte Carlo methods with an analytical reduction of the defining multidimensional integral.

Radial symmetry and Fourier/Hankel-transform techniques lead to an explicit and computationally efficient statistic, while characteristic-function algorithms developed within the CharFunTool framework support multivariate distributional calculations, simulation, and visualization. The procedure is affine invariant, its asymptotic behaviour is established under the null hypothesis and contiguous alternatives, and finite-sample critical values are obtained by Monte Carlo calibration.

Simulation studies show accurate size and strong power relative to multivariate Kolmogorov–Smirnov, energy, and chi-square procedures, particularly under heavy-tailed and higher-dimensional alternatives. Applications to bivariate educational test scores and four-dimensional Iris measurements illustrate practical model validation. The presentation emphasizes a broader message: characteristic functions can serve not only as theoretical descriptors of distributions, but also as an effective computational route from probabilistic modelling and numerical distribution theory to implementable multivariate statistical inference.

Keywords

Characteristic functions, CharFunTool, numerical inversion, empirical characteristic function, goodness-of-fit, multivariate logistic distribution.

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References

[1] Mijanović, A., Popović, B.V. and Witkovský, V. (2023). A numerical inversion of the bivariate characteristic function. *Applied Mathematics and Computation 443*, 127807.

[2] Popović, B.V., Mijanović, A. and Witkovský, V. (2026). A goodness-of-fit test for multivariate logistic distributions. *Statistics*. DOI: 10.1080/02331888.2026.2624510.

[3] Witkovský, V. (2020). *CharFunTool: The Characteristic Functions Toolbox (MATLAB)*. GitHub repository: https://github.com/witkovsky/CharFunTool/.