

Sharp bounds on L -moments

Tomasz Rychlik¹ and Magdalena Szymkowiak²

¹*Institute of Mathematics, Polish Academy of Sciences, Poland*

²*Poznań University of Technology, Poland*

Abstract

The expectations of specific linear combinations of order statistics from i.i.d. samples with a finite expectation parent distribution are called L -moments and denoted as λ_r , $r = 1, 2, \dots$. They were introduced by Hosking [1]. The L -moments uniquely characterize the distribution, and admit to explicitly recover its quantile function. The first L -moments λ_r , $r = 1, 2, 3, 4$, describe basic shape properties of the distribution: location, dispersion, skewness, and peakedness, respectively. We present sharp lower and upper bounds on L -moments expressed in the scale units based on central absolute moments of the baseline distribution, and the second L -moment being a half of the Gini mean difference. We also determine the distributions attaining these bounds. The talk is based on papers [2] and [3].

Keywords

L -moment, Sharp bound, Attainability condition.

References

- [1] Hosking, J.R.M. (1990). L -moments: analysis and estimation of distributions using linear combinations of order statistics. *J. R. Stat. Soc. Ser. B Stat. Methodol.* 52, 105–124.
- [2] Rychlik, T., Szymkowiak, M. (2026). Extreme values of scaled L -moments. *Statist. Probab. Lett.* 231, Paper No. 110626, 6 pp. doi: 10.1016/j.spl.2025.110626
- [3] Rychlik, T., Szymkowiak, M. (2026). Optimal bounds for L -moments, under review.