Stochastic dominance between linear combination

Tommaso Lando[1] **and Paulo Eduardo Oliveira**[2]

[1]*Department of Economics, University of Bergamo, via dei Caniana 2, 24127, Bergamo, Italy*
[2]*CMUC, Department of Mathematics, University of Coimbra, Portugal*

Abstract

Convex combinations of independent and identically distributed random variables without a finite mean can behave in a strikingly different way from the finite-mean case. First results proved the stochastic dominance of convex combinations with respect to the underlying distribution [2], and fostered the interest in enlarging the class of distributions for which the dominance holds and in finding tractable conditions implying this dominance [4, 1]. The more general comparison between arbitrary combinations of variables was first discussed in [3], identifying a large class of distribution and a suitable majorization relation between the combination coefficients such that the stochastic dominance still holds.

We discuss a relaxation of the class of distributions that implies the dominance between convex combinations. Moreover, for the specific case of sample means, we discuss some distribution free results that also allow to derive dominance relations. From the application point of view, we propose a Kolmogorov-Smirnov one-sample type of test to decide upon this dominance, characterising its asymptotic properties and providing some insight on its numerical performance.

Keywords

Bootstrap, Empirical process, Infinite mean, Majorization, Nonparametric test, Sample mean, Stochastic order.

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