Dependence Reduction by Linear Residualization

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Abstract

A standard way to reduce dependence between two random variables is to regress one on the other and work with the resulting residual. While this procedure removes linear correlation by construction, its effect on nonlinear dependence is much less clear. We study this question in a copula framework. Starting from a bivariate vector (U, V) with copula C, we consider monotone marginal transformations $Z_1 = H_1^{-1}(U)$ and $Z_2 = H_2^{-1}(V)$ and the residualized variable $\varepsilon_\rho = Z_2 - \rho Z_1,\ \rho \in [-1, 1]$. We derive the copula of (Z_1, ε_ρ) and investigate how residualization changes measures of dependence, with particular emphasis on Kendall's tau.

We show that the Kendall's tau of (Z_1, ε_ρ) is non-increasing in ρ, strictly so under mild regularity conditions. This yields conditions under which residualization moves dependence towards independence and provides bounds on the admissible residualization parameter. In the Gaussian case, choosing ρ equal to the correlation coefficient eliminates both Pearson correlation and Kendall dependence. We demonstrate, however, that this property is genuinely Gaussian: outside the Gaussian copula family, eliminating linear correlation need not eliminate rank dependence and can even reverse its sign. Explicit counterexamples based on mixtures of Gaussian copulas and asymmetric copulas illustrate this phenomenon. We further propose empirical estimators of the residualized copula and Kendall's tau and establish uniform consistency with respect to ρ. Finally, the framework is extended to jointly select the marginal transformations and the residualization parameter so as to minimize a chosen measure of dependence.

Keywords

Copula, dependence reduction, residualization, Kendall's tau, nonlinear dependence, empirical copula.