Testing hypotheses on covariance matrices within linear structure spaces

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Abstract

The problem of testing whether the true covariance matrix belongs to a given non-quadratic linear space of symmetric matrices is considered. This distinction matters because, as shown by [1] and [2], the MLE reduces to an orthogonal projection of the sample covariance matrix only in the quadratic case; for non-quadratic subspaces, no closed form solution exists, and numerical optimization is required. To overcome this difficulty, two alternative estimators are proposed: the restricted maximum likelihood (RML) estimator, obtained via numerical optimization, and a shrinkage structured (SH) estimator and use them in place of the standard MLE within the classical likelihood ratio test (LRT) and Rao score test (RST).

Our main theoretical contribution establishes that the LRT statistics based on the RML and SH estimators retain the same asymptotic chi-squared distribution as their classical MLE-based counterparts. A simulation study is conducted to examine the finite sample behavior of both the LRT and RST statistics: for the LRT, this includes verifying the rate of convergence to the limiting distribution, while for the RST, the chi-squared distribution is used as an empirical reference. The study further compares the power of the tests and the computation times associated with each estimator.

The proposed methodology is further illustrated through an application to real data, where model selection criteria are used to determine the most appropriate covariance structure under each method.

Keywords

Covariance matrix, Testing hypotheses, Linear structures, Toeplitz matrix, Shrinkage estimator, Restricted maximum likelihood estimator.

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References

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