On the combinatorics of generalized cumulants and k-statistics

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Abstract

Generalized cumulants offer a flexible framework for the analysis of polynomial functions of random variables [3]. They can be viewed as intermediate objects between joint moments and joint cumulants, providing a structured way to compute the cumulants of such functions.

Within this framework, k-statistics and their extensions yield unbiased estimators of generalized cumulants. While the classical theory provides a well-established framework for ordinary cumulants and k-statistics, its extension to generalized cumulants presents additional challenges, primarily due to the complexity involved in enumerating complementary set partitions required for their computation. This complexity can limit computational efficiency and practical implementation in non-symbolic numerical environments.

In this talk, we present a novel combinatorial approach for generating complementary set partitions based on two-block partitions, leading to improved computational efficiency and enabling implementation in non-symbolic environments [2]. We also extend generalized cumulants to products of random variables indexed by multiset subdivisions, allowing for repetitions and powers.

A key ingredient is the use of multi-index partitions, which provide a tractable framework for handling multivariate cumulant formulas and for constructing multivariate polykays [1]. The results highlight the interplay between combinatorial techniques and statistical methodology, offering both theoretical insight and practical tools for cumulant-based analysis.

Keywords

Complementary set partitions, Multivariate polykays, Multi-index partitions, Combinatorial algorithms.

References

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