

On copula-based systems of quantile curves with application in outliers detection

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Abstract

Systems of curves representing quantiles of a bivariate distribution are introduced. Such systems consist of decreasing curves that partition the support of the distribution into nested subsets, each associated with a prescribed probability, which can be interpreted as the quantile level corresponding to the respective curve.

For continuous bivariate distributions, there exist uncountably many systems of quantile curves. Some approaches to defining such systems have been proposed, among others, by [1] and [2]. Here, we focus on a particular class of systems based on the copula associated with the underlying bivariate distribution. The use of the copula makes it possible to construct quantile curves in two steps. First, a system of quantile curves is identified on the support of the copula (the unit square). These curves are then mapped onto the support of the original bivariate distribution through the marginal distributions, yielding a system of copula-based quantile curves.

Some preliminary inferential results are also presented, allowing the construction of both pointwise and simultaneous confidence bands for the quantile curves.

Finally, a first attempt at using the proposed notion of quantile curves for outlier detection in bivariate data is presented.

Keywords

multivariate distributions, multivariate quantile, copula function.

References

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