Moving Random Fields Constructing and Estimating Motion in Spatio-Temporal Random Fields

Anastassia Baxevani

University of Cyprus

Abstract

Many environmental processes - precipitation, ocean waves, wind fields - are not merely spatially correlated but move: weather systems drift, grow, and decay coherently in time. Capturing this motion requires more than a spatial random field evolving pointwise in time; it requires a model with a genuine underlying velocity. This talk presents a framework for constructing and estimating such moving random fields. We first formalize what it means for a random field to move, using excursion sets, crossing contours, and a generalized Rice formula that characterizes the distribution of the field's velocity at a level crossing. Building on this, we construct Gaussian random fields driven by a deterministic flow: first advected by a constant velocity, then generalized to a spatially and temporally varying velocity field via a flow of diffeomorphisms driving a stochastic integral. A key theorem shows that, under isotropic innovations, the field's own random velocity distribution is centered exactly on the deterministic flow used to construct it - confirming the construction behaves as intended. We then discuss two complementary strategies for estimating the velocity field from data: a method-of-moments approach exploiting structure in the spatio-temporal covariance, and a state-space formulation solved via particle filtering. We close by showing that the resulting motion signature is directly visible in the estimated covariance of real precipitation data.