

The Greenwood Statistic

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Abstract

Let $X_1, X_2, \dots, X_n$ be an i.i.d. random sample from a distribution supported on $[0, \infty)$. The Greenwood statistic is defined by

$$T_n = \frac{\sum_{i=1}^n X_i^2}{\left(\sum_{i=1}^n X_i\right)^2}.$$

This talk reviews a series of results on the Greenwood statistic and its modifications. First, we discuss how star-shaped ordering of the underlying distributions leads to stochastic ordering of the Greenwood statistic, providing a basis for inference in heavy-tailed models [1]. In particular, for the generalized Pareto distribution, this approach leads to hypothesis tests for the tail index [2]. We then introduce a modified Greenwood statistic for symmetric distributions, with applications to Gaussianity testing and inference for symmetric α -stable and Student's t distributions [3, 4]. Finally, we discuss multivariate extensions applied to sub-Gaussian, multivariate Pareto, and multivariate Student's t models [5]. These results illustrate the versatility of the Greenwood statistic and its modifications as simple tools for statistical inference in heavy-tailed and non-Gaussian settings.

Keywords

stochastic orders, clustering, generalized Pareto distribution, heavy-tailed distributions, Gaussianity testing.

References

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